Answers to Larmor Precession Tutorial

(only check these after you are finished with the complete tutorial)

1. C.
2. B.
3. A. Andy
4. 1 (eigenstate of $\hat{H}$)
5.
6. No
7.
8. $|\chi(t)\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle_x + |\downarrow\rangle_x\big)\, e^{i\gamma B_0 t/2}$
9. 1/2
10.
11. No (still the same eigenstate of $\hat{H}$)
12.
13. $\langle \hat{S}_x \rangle = 0$ (at all times)
14.
15.
16. D.
17. C.
18. Yes; $a = b = 1/\sqrt{2}$, $\alpha = \pi/4$, $\phi_1 = \phi_2 = 0$
19. C.
20. 1/2
21.
22. No
23.
24. $|\chi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{i(\phi_1+\gamma B_0 t/2)}\cos\alpha + e^{i(\phi_2-\gamma B_0 t/2)}\sin\alpha\right)|\uparrow\rangle_x + \frac{1}{\sqrt{2}}\left(e^{i(\phi_1+\gamma B_0 t/2)}\cos\alpha - e^{i(\phi_2-\gamma B_0 t/2)}\sin\alpha\right)|\downarrow\rangle_x$

25. B.

26. probability is $\cos^2(\gamma B_0 t/2)$; yes, it depends on time;
$\cos^2(\pi/6) = \frac{3}{4}$, $\cos^2(\pi/4) = \frac{1}{2}$, $\cos^2(\pi/2) = 0$

27. No, the state evolves into a superposition of $|{\uparrow}\rangle_x$ and $|{\downarrow}\rangle_x$, since $|{\uparrow}\rangle_x$ is not an eigenstate of $\hat{H}$

28. $\langle S_x(t)\rangle = \cos(\gamma B_0 t)\,\hbar/2$
$\langle S_x(t_0)\rangle = \cos(\pi/3)\,\hbar/2 = \frac{1}{2}\,\hbar/2$

29. $\frac{1}{2}\,\hbar/2 = \left(\frac{3}{4} - \frac{1}{4}\right)\,\hbar/2$

30. probability is $\left|\frac{1}{2}\left(e^{-i\pi/4} - ie^{+i\pi/4}\right)\right|^2 = 1$

31.

32. $\hat{S}_z$ commutes with $\hat{H}$, but $\hat{S}_x$ and $\hat{S}_y$ do not

33. Mira.

34. C.

35. D.

36. A.

37.

38. $|{\downarrow}\rangle_z$ is a stationary state of $\hat{H}$, so all expectation values are time-independent.
$\frac{1}{\sqrt{2}}\left(|{\uparrow}\rangle_z + |{\downarrow}\rangle_z\right)$ is not a stationary state. Still, the expectation values of all observables that commute with $\hat{H}$ do not depend on time. In our case this is true for $\hat{S}_z$, but not for $\hat{S}_x$ and $\hat{S}_y$.